Exponential suppression of the topological gap in self-consistent intrinsic Majorana nanowires

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Motivation

In pursuit of

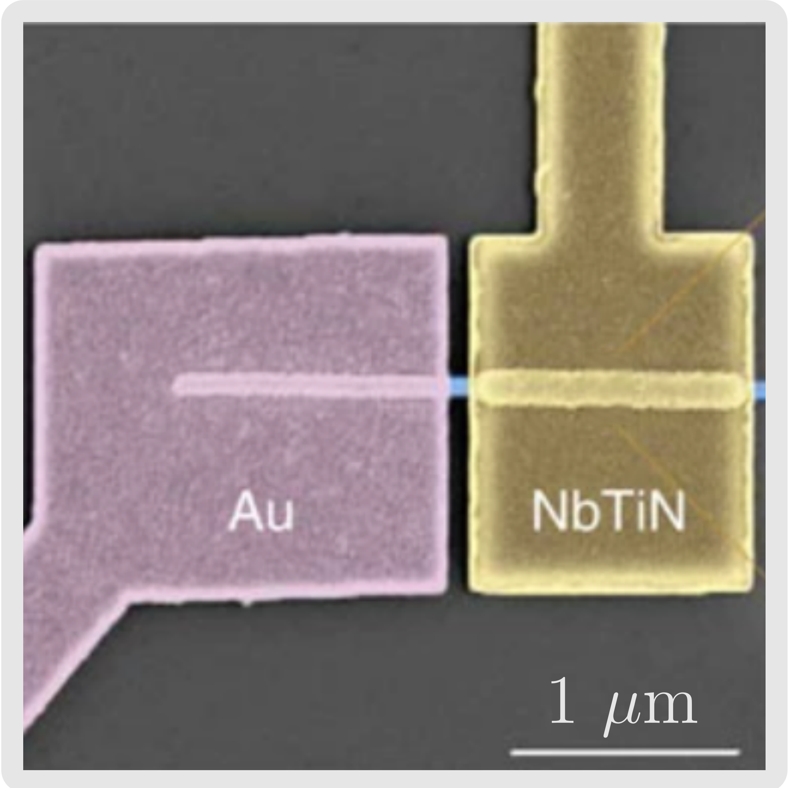
Fault-tolerant topological qubits

Platforms that may be viable

Majorana hybrid nanowires

Experimental struggles

- smooth confinement
- trivial state pinning
- metalization
- disorder



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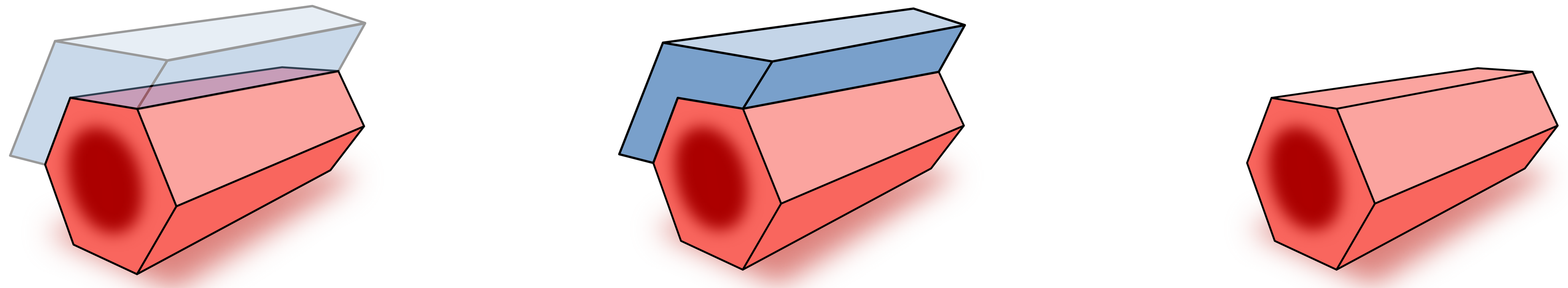
Introduction

Alternative platform that bypasses hybrid complexities

Nearly-depleted nanowires with intrinsic superconductivity

(as opposed to proximity-induced)



Theoretical background

Self-consistent Hartree-Fock-Bogoliubov mean-field theory

Self-consistent superconductivity

Hubbard model

$$H^{\mathrm{Hub}} = H_0 + H_U$$

site j site i

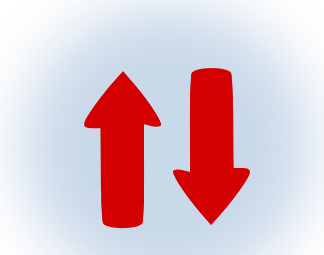
$U<0$

Kinectics

On-site attraction

BCS mean-field theory

$$\Sigma_{\mathrm{BCS}} = \sum_i \underbrace{U \left\langle c_{i\uparrow} c_{i\downarrow} \right\rangle}\, c^{\dagger}_{i\downarrow} c^{\dagger}_{i\uparrow} + \mathrm{h.c.}$$



Cooper pair

anomalous order parameter

i.e superconducting pairing Δ^{ii}

Hartree-Fock-Bogoliobouv mean-field theory

Nambu doubling

$$\check{c}^{\dagger}_i = \begin{pmatrix} c^{\dagger}_{i\uparrow} & c^{\dagger}_{i\downarrow} & c_{i\uparrow} & c_{i\downarrow} \end{pmatrix}$$

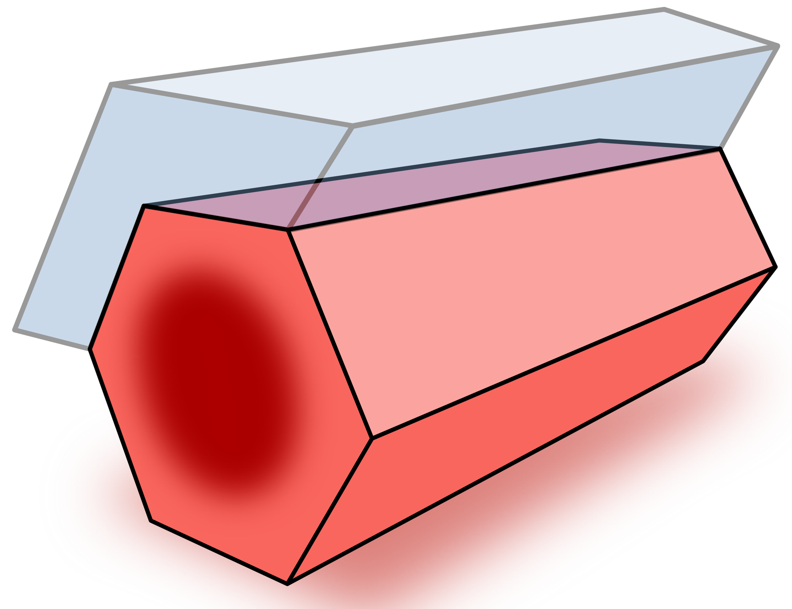
$\rho^{ii}_{e\sigma,e\sigma'} = \langle c^{\dagger}_{i\sigma'} c_{i\sigma} \rangle$ and $\rho^{ii}_{h\sigma,e\sigma'} = \langle c_{i\sigma'} c_{i\sigma} \rangle$

$$\Sigma_{\mathrm{HFB}} = \frac{1}{2} \sum_{i\sigma\sigma'} (c^{\dagger}_{i\sigma}, c_{i\sigma}) \check{\Sigma}^{ii}_{\sigma\sigma'} \begin{pmatrix} c_{i\sigma'} \\ c^{\dagger}_{i\sigma'} \end{pmatrix}$$

$$\check{\Sigma}^{ij} = U \delta_{ij} \left(\frac{1}{2} \mathrm{Tr}(\tau_z \tilde{\rho}^{ii}) \tau_z - \tau_z \tilde{\rho}^{ii} \tau_z \right)$$

Nambu symmetrized rDM

Oreg-Lutchyn majorana nanowire

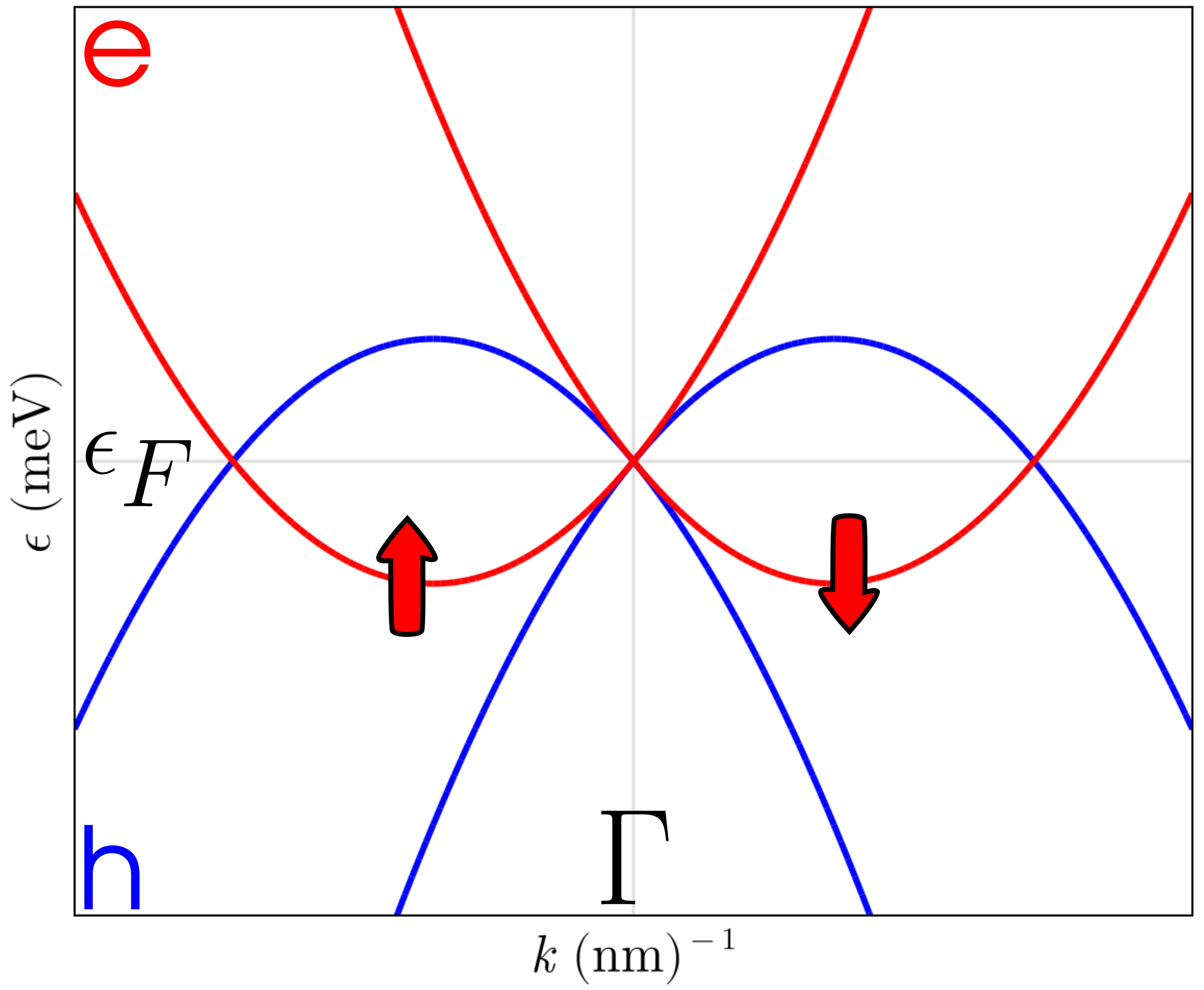


super
(integrated out)
semi
fixed $\Delta^{ii} \neq 0$

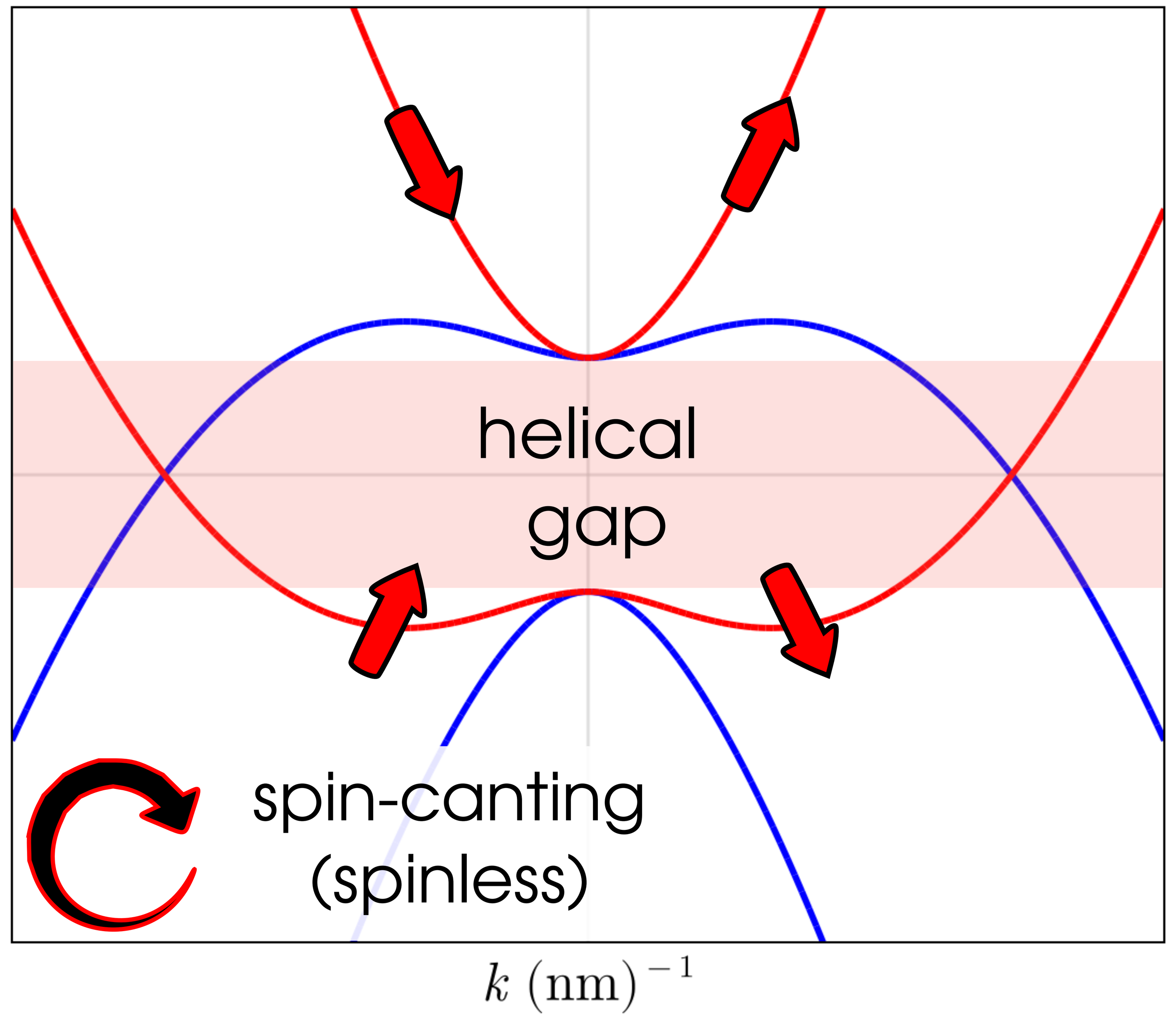
Kinectics Rashba Zeeman Pairing

$$H^{\mathrm{OL}} = H_0 + H_{\mathrm{SOC}} + H_{\mathrm{Z}} + H_{\Delta^{ii}}$$

SOC

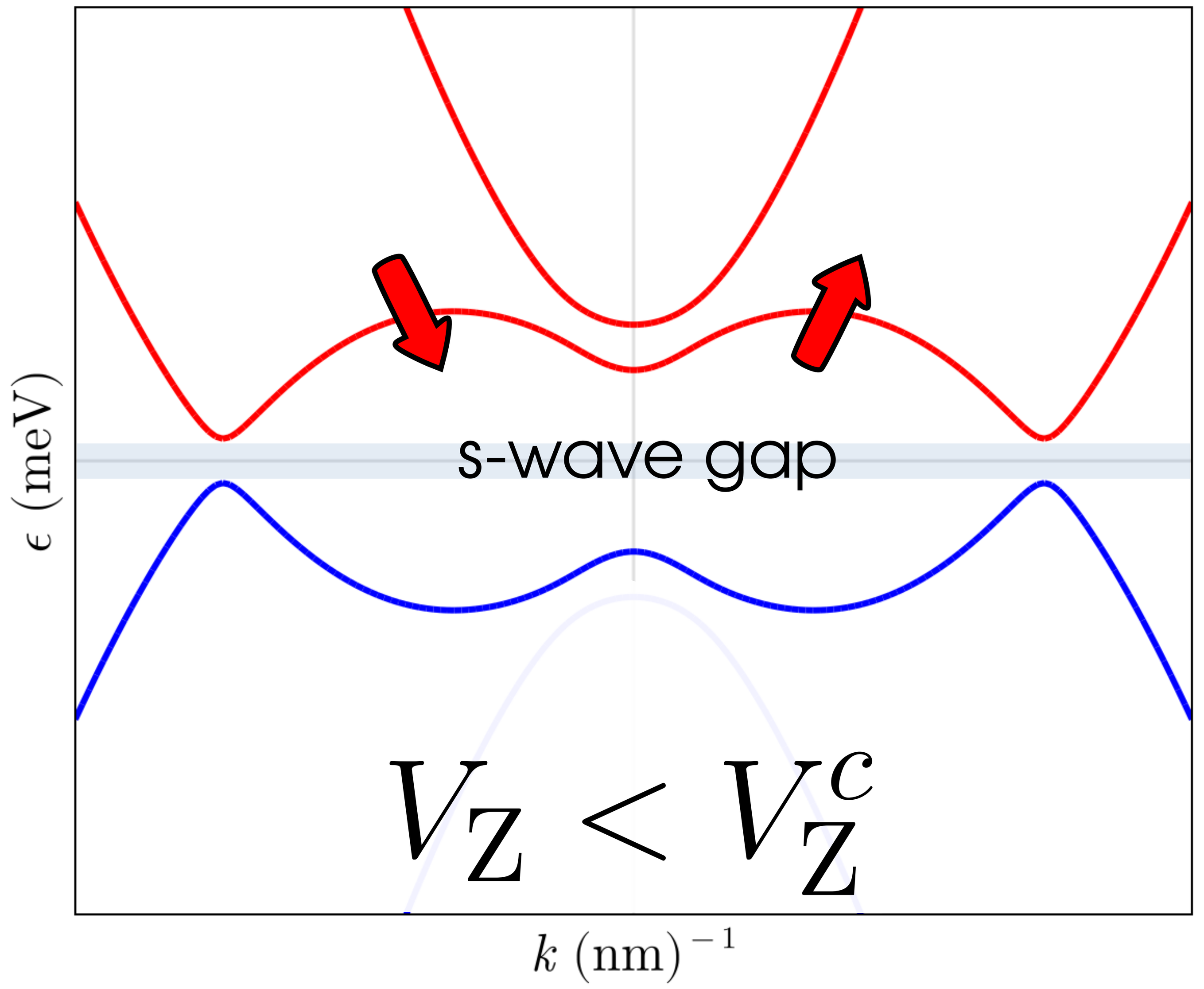


SOC + Zeeman

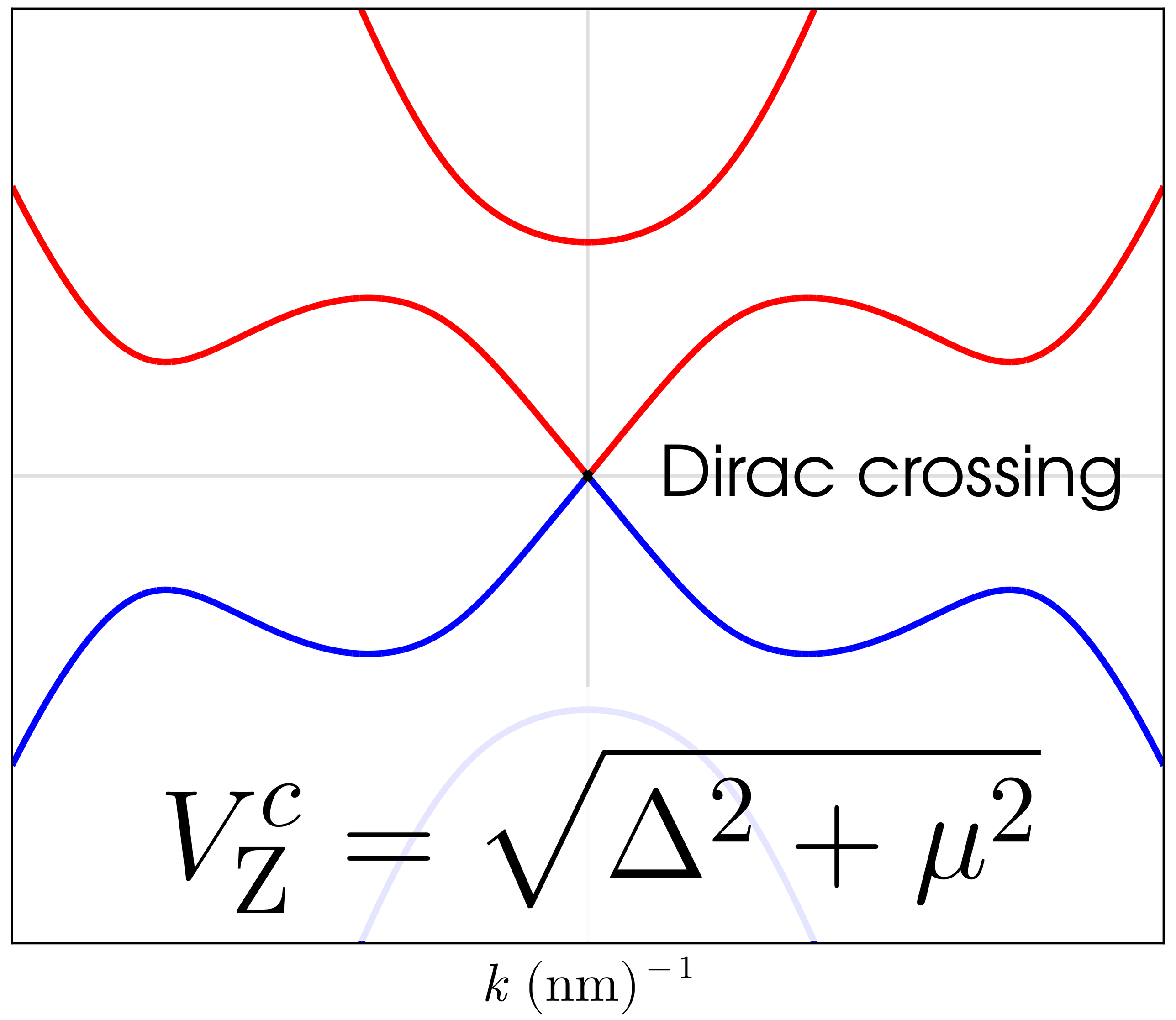


SOC + Zeeman + pairing phase transitions

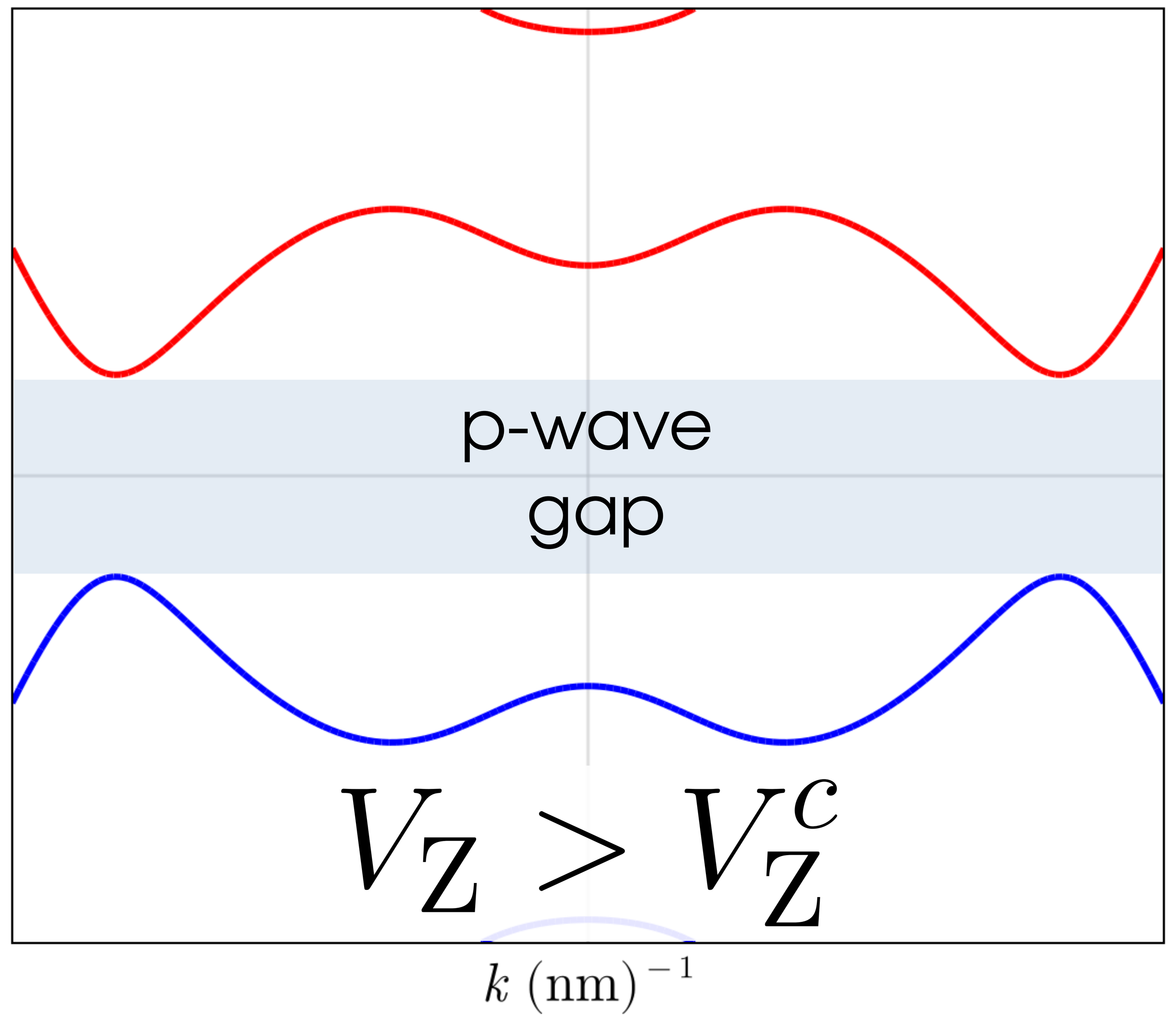
Trivial



Critical

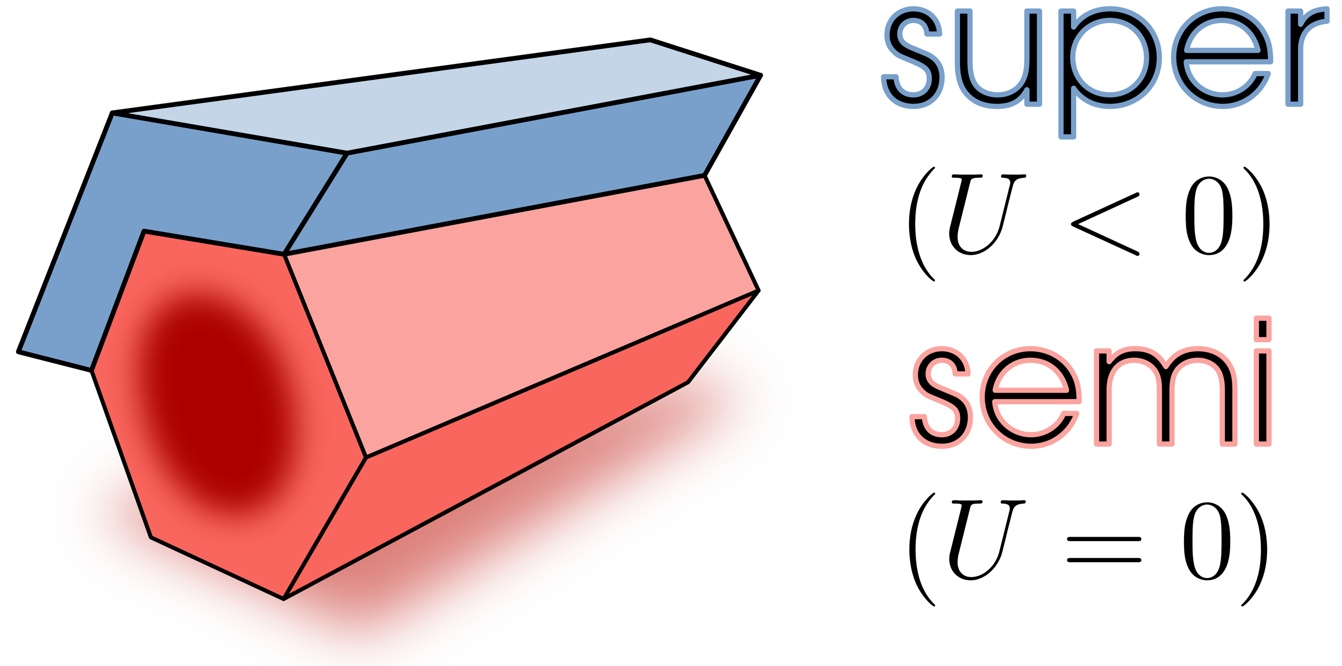


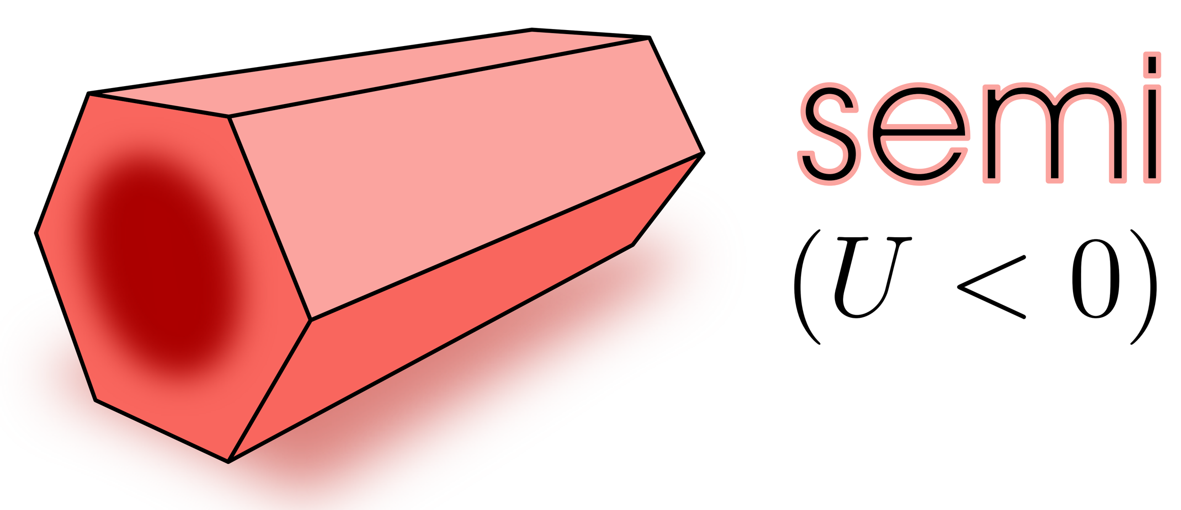
Topological



Self-consistent majorana nanowires

hybrid v.s intrinsic



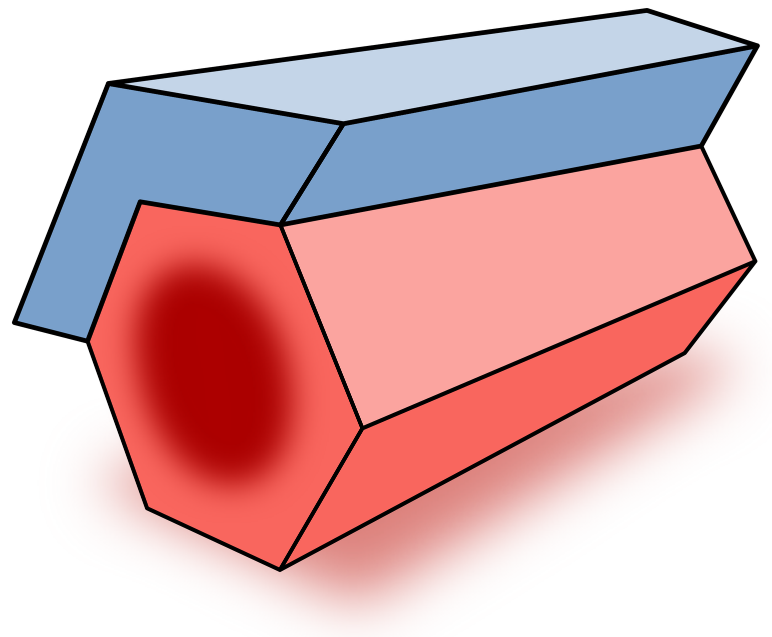


$$H^{\mathrm{hyb}} = H_0^{\mathrm{SM}} + H_{\mathrm{Z}}^{\mathrm{SM}} + H_{\mathrm{SOC}}^{\mathrm{SM}} + H_0^{\mathrm{SC}} + H_{\mathrm{Z}}^{\mathrm{SC}} + \Sigma_{\mathrm{HFB}}^{\mathrm{SC}} + H^{\mathrm{SC\text{-}SM}}$$

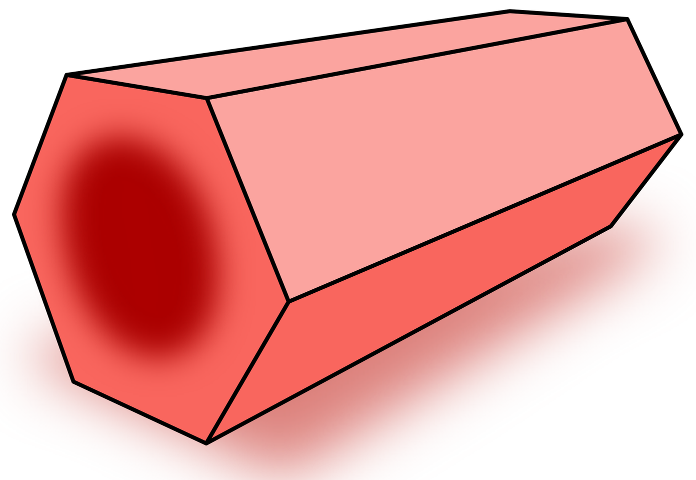
$$H^{\mathrm{int}} = H_0 + H_{\mathrm{SOC}} + H_{\mathrm{Z}} + \Sigma_{\mathrm{HFB}}$$

Self-consistent majorana nanowires

hybrid v.s intrinsic



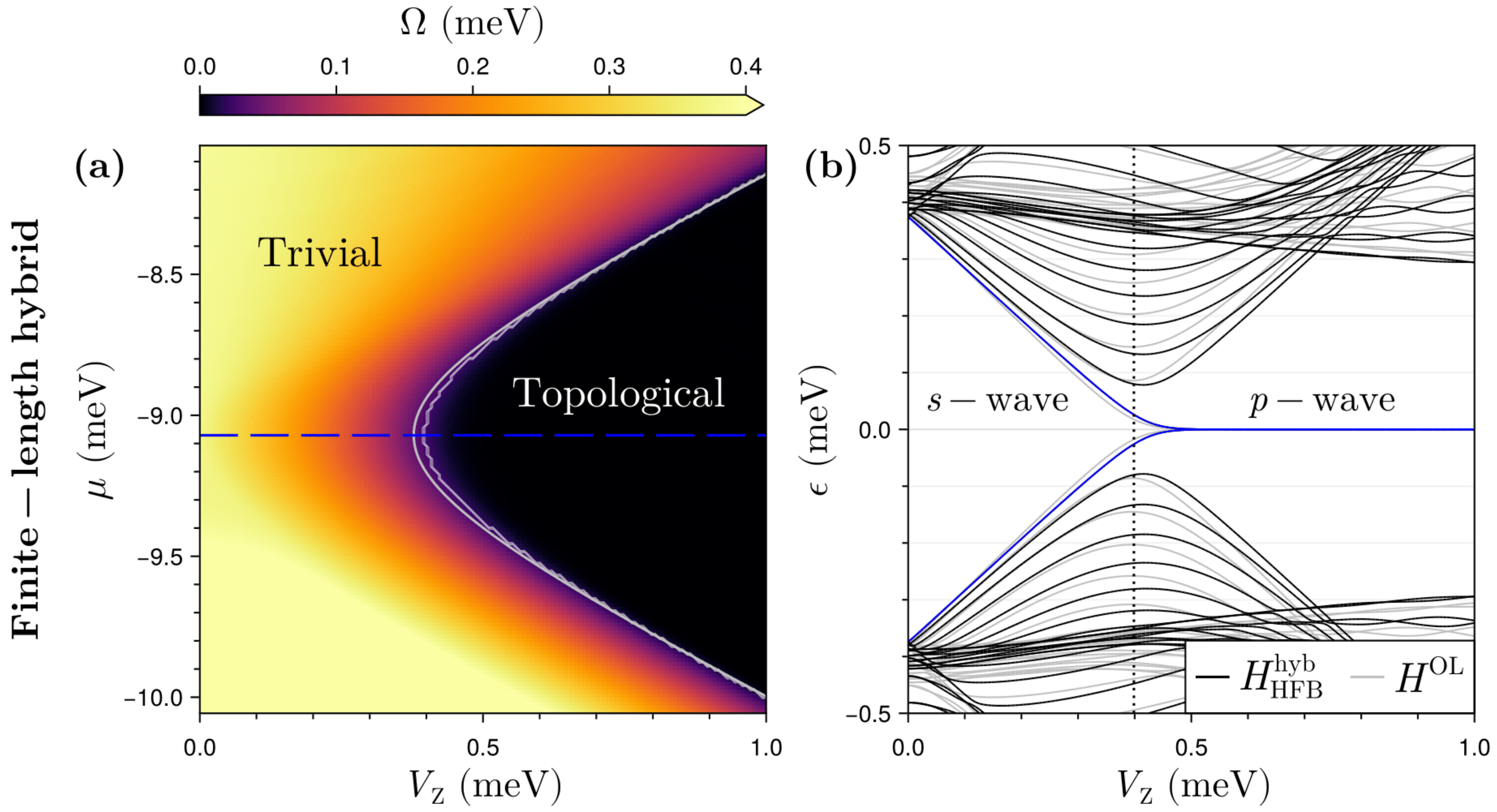
super
$(U < 0)$
semi
$(U = 0)$



semi
$(U < 0)$

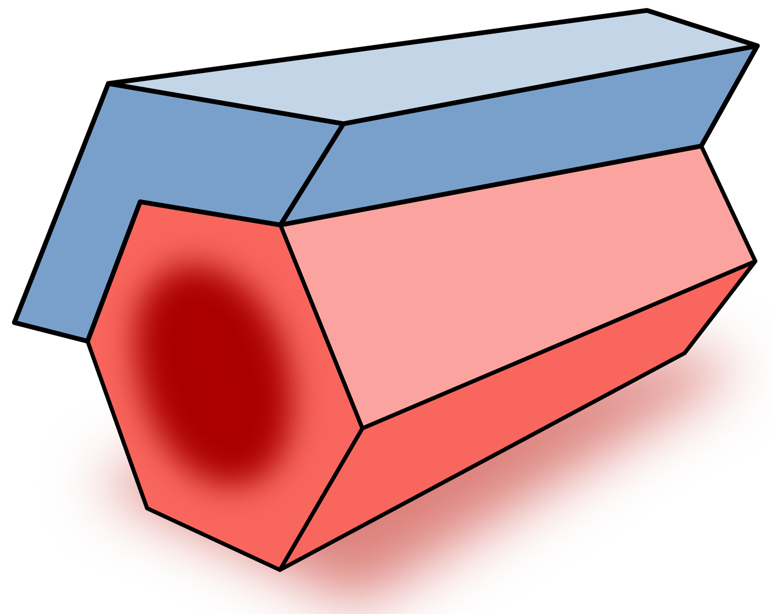
$$H^{\text{hyb}} = H_0^{\text{SM}} + H_{\text{Z}}^{\text{SM}} + H_{\text{SOC}}^{\text{SM}} + H_0^{\text{SC}} + H_{\text{Z}}^{\text{SC}} + \Sigma_{\text{HFB}}^{\text{SC}} + H^{\text{SC-SM}}$$

$$H^{\text{int}} = H_0 + H_{\text{SOC}} + H_{\text{Z}} + \Sigma_{\text{HFB}}$$



Self-consistent majorana nanowires

hybrid v.s intrinsic



super
$(U < 0)$
semi
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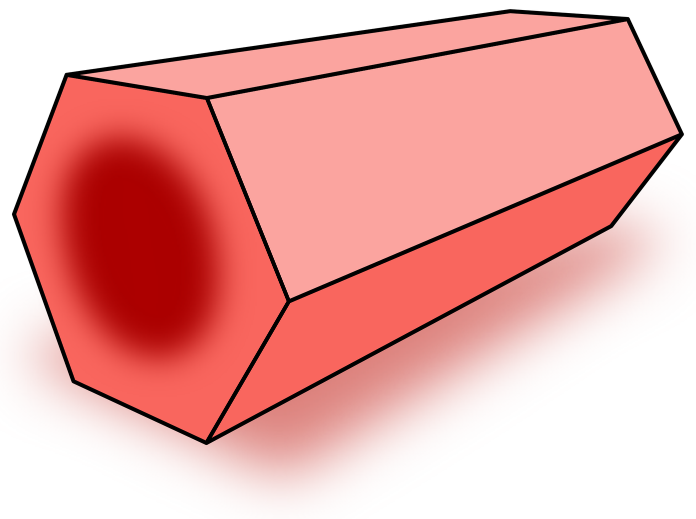
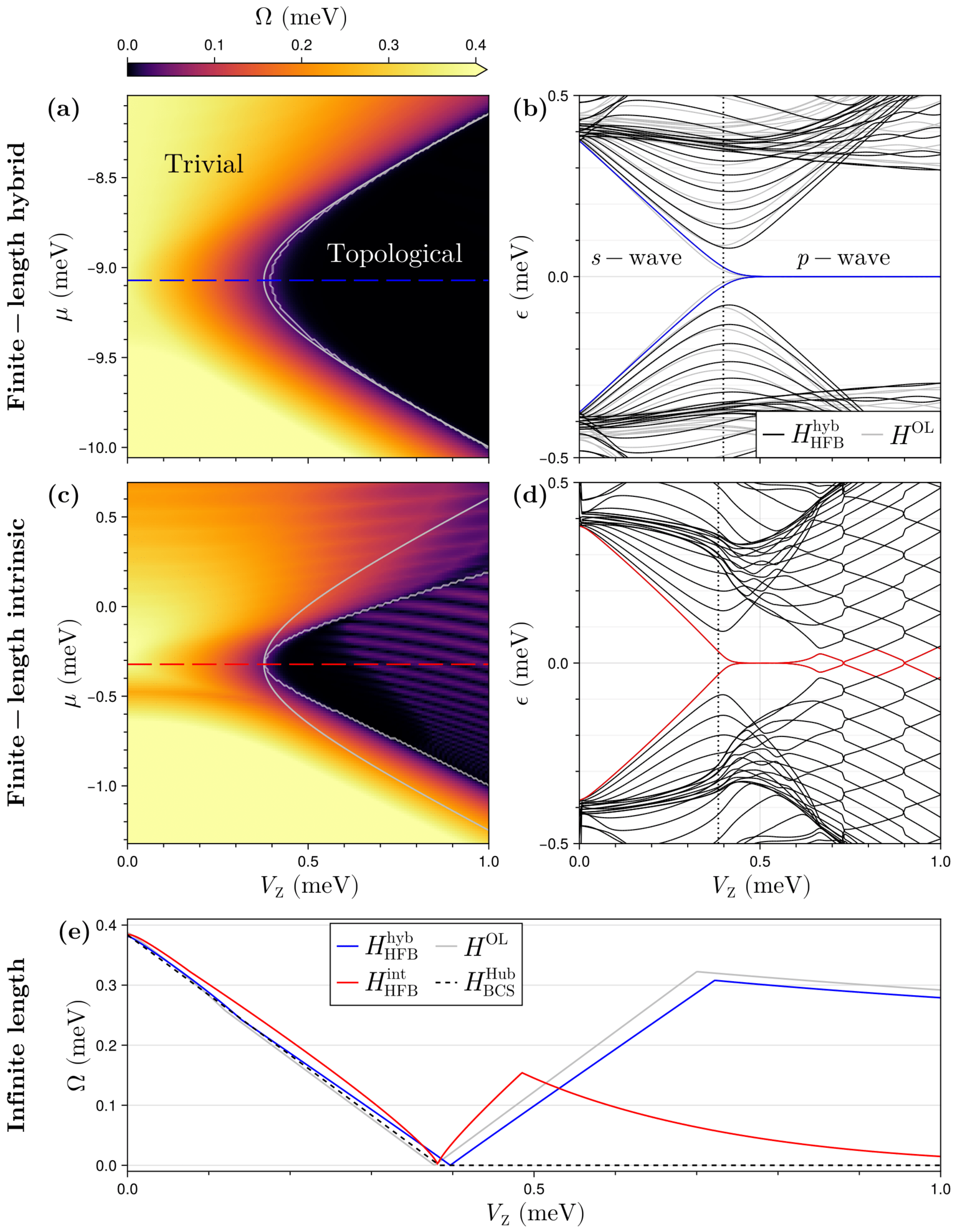


semi
$(U < 0)$

$$H^{\text{hyb}} = H_0^{\text{SM}} + H_Z^{\text{SM}} + H_{\text{SOC}}^{\text{SM}} + H_0^{\text{SC}} + H_Z^{\text{SC}} + \Sigma_{\text{HFB}}^{\text{SC}} + H^{\text{SC-SM}}$$

$$H^{\text{int}} = H_0 + H_{\text{SOC}} + H_Z + \Sigma_{\text{HFB}}$$

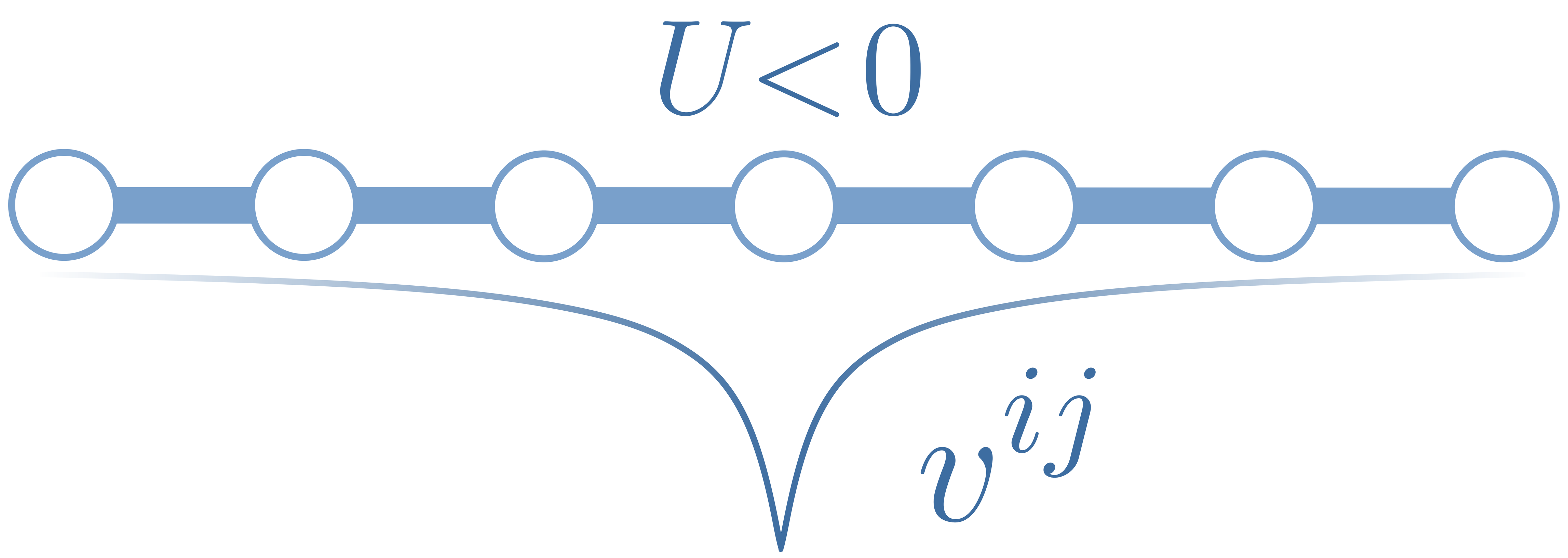


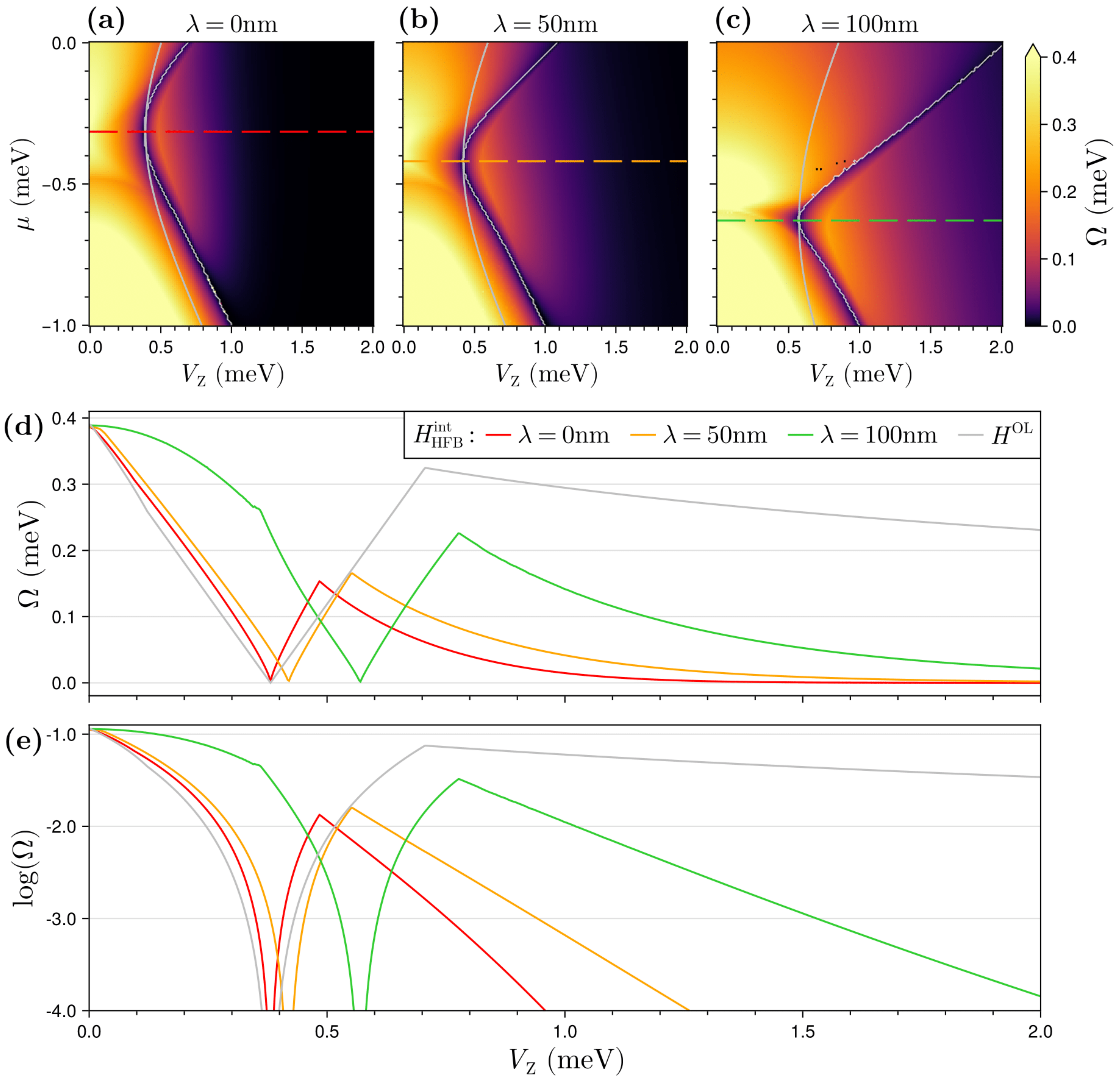
Intrinsic model w/ finite range interactions

$$H_U \longrightarrow H_{\text{int}} \text{ for } i \neq j$$

with a screened Coulomb interaction of screening length λ

(imposed onsite Hubbard still)



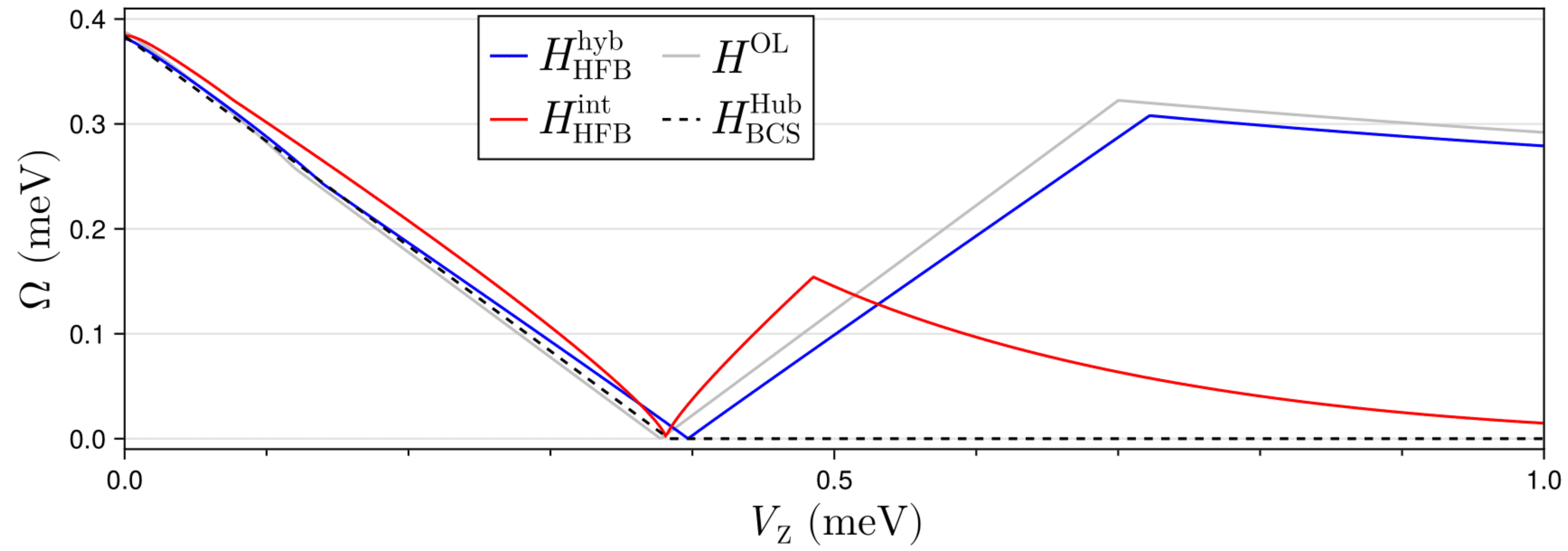


Conclusions

Are nearly-depleted nanowires with intrinsic superconductivity promising as a substitute to their hybrid counterpart?

Not really. Still a problematic approach due to:

- exponential decay of the minigap with Zeeman
  - spinlessness versus superconductivity
  - weak spin canting of spinless carriers*



But at least the developed self-consistent superconductivity methodology can prove to be fruitful for other systems

* does not affect Majoranas obtained by the time-reversal symmetric Fu-Kane approach. DOI: 10.1103/PhysRevLett.100.096407

Thank you for listening!
Any questions?

arXiv:2412.15174